Fakultät Informatik/Mathematik

HOCHSCHULE FÜR
TECHNIK UND WIRTSCHAFT
DRESDEN
UNIVERSITY OF APPLIED SCIENCES

DEPENDIABILITY OF COMPUTER SYSTEMS

BELEGARBEIT, VARIANTE 5

Beleg von

Falk-Jonatan Strube (s74053)

Betreuender Hochschullehrer

Prof. Dr.-Ing. habil. Peter Sobe

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INHALTSVERZEICHNIS

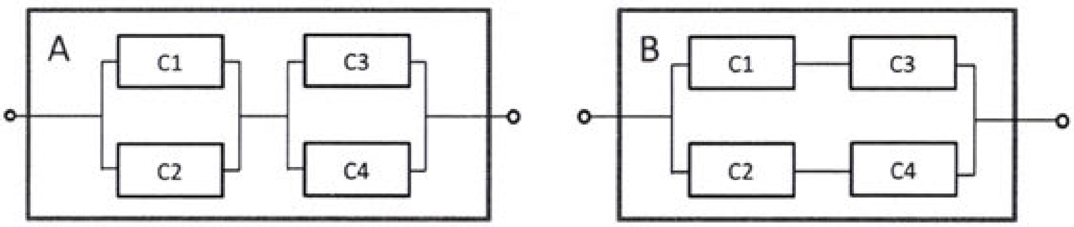
AUFGABEN

1: [2 points]

Assume a computer node with a reliability R=0.99999 for a fixed mission time.
What is the maximum number of nodes that can be composed as a serial system, e.g. by running a parallel program cooperatively on that number of nodes, when the total failure probability may not exceed 0.0005?

2: [4 points]

Assume two systems, A and B that are described by the reliability block diagrams below. The components are denoted by C1, C2, C3 and C4.
The reliability of all components is equal: $R_{C1}=R_{C2}=R_{C3}=R_{C4}=R$



Q1: Which system (A or B) is more reliable?

Q2: Show that the decision from Q1 holds for every value of R, $0< R< 1$!

3: [4 points]

A RAID level 6 disk array contains k disks, and in addition m disks for redundant data that is calculated by an error correcting code. The code allows a recalculation of all data items on the array, as long as no more than m disks are failed simultaneously. In case of repair, the code allows to reconstruct data on a device that failed and gets replaced by an operational one.

All disks are assumed to fail with the rate λ=0.000001 per day. Repair of a single disk is completed within 2 days (repair rate μ=0.5 per day).
Particularly, the reliability of a k=4, m=2 array (4 of 4+2 system) has to be evaluated for a mission time of 1 year.

Q1: Construct a Markov model for the case that repair is not applied!
Classify states that cover an operational system, and the state(s) that describe a failed system!

Q2: Construct the corresponding Markov model including repair!
Classify states that cover an operational system, and the state(s) that describe a failed system!

The numeric values of R and F can be obtained using the Markov model simulator (mm_solve). The numeric values are not required for the exam. However, the numbers and the model files for mm_solve are welcome and are credited by extra points when correct.

1 AUFGABE 1

x ... Anzahl der Computerknoten in einem seriellen System.
Berechnung der Anzahl, die nicht überschritten werden darf:

$$\begin{aligned} 1-(0,99999)^x &\geq 0,0005 \\ (0,99999)^x &\leq 0,9995 && \mid \log(...) \\ x \cdot \log(0,99999) &\leq \log(0,9995) \\ x &\leq \frac{\log(0,9995)}{\log(0,99999)} \\ x &\leq 50,01225411 \end{aligned}$$

Antwort: Die maximale Anzahl von Computerknoten beträgt 50.

2 AUFGABE 2

Q1: Das System A ist zuverlässiger.
Wenn eine Komponente ausfällt sind beide System noch läuffähig.
Bei zwei Komponenten dürfen bei System A 4 Kombinationen ausfallen (die „Diagonalen“ [*C1 und C4* und *C2 und C3*] und die „Senkrechten“ [*C1 und C3* und *C2 und C4*]). Bei System B dürfen nur die „Senkrechten“ (also nur zwei Kombinationen) gleichzeitig ausfallen.
Wenn drei Komponenten ausfallen sind beide Systeme nicht mehr lauffähig.

Q2: Gesamtzuverlässigkeit:
System A, zwei parallele Komponenten in Reihe:

$$R_A = \underbrace{(\underbrace{1-(1-R)^2}_{\text{Parallel}})^2}_{\text{Reihe}}$$

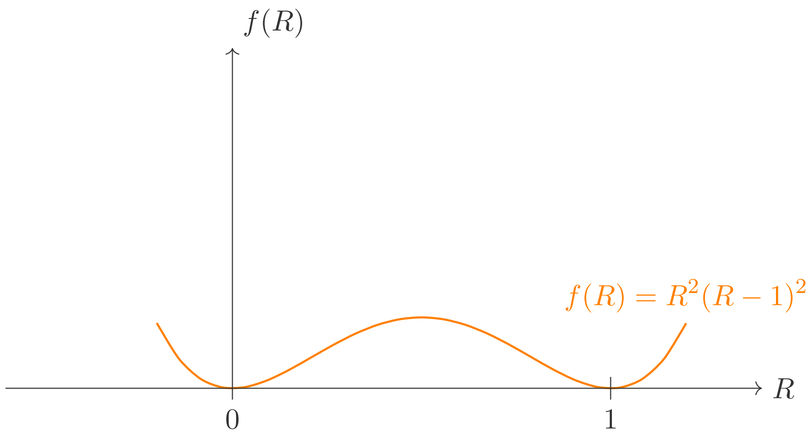
System B, zwei Komponenten in Reihe parallel zueinander:

$$R_B = \underbrace{1-(1-\underbrace{R^2}_{\text{Reihe}})^2}_{\text{Parallel}}$$

Zu prüfen ist, ob Zuverlässigkeit R_A stets größer ist als R_B (mit $0 < R < 1$):

$$\begin{aligned} R_A &> R_B \\ (1-(1-R)^2)^2 &> 1-(1-R^2)^2 \\ 1-2(1-R)^2+(1-R)^4 &> 1-(1-2R^2+R^4) && \mid -1 \\ (1-R)^4-2(1-R)^2 &> -1+2R^2-R^4 \\ (1-2R+R^2)^2-2(1-2R+R^2) &> -1+2R^2-R^4 \\ 1-4R+6R^2-4R^3+R^4-2+4R-2R^2 &> -1+2R^2-R^4 \\ -1+4R^2-4R^3+R^4 &> -1+2R^2-R^4 && \mid +1-2R^2+R^4 \\ 2R^2-4R^3+2R^4 &> 0 && \mid /2 \\ R^2(1-2R+R^2) &> 0 \\ R^2(R-1)^2 &> 0 \end{aligned}$$

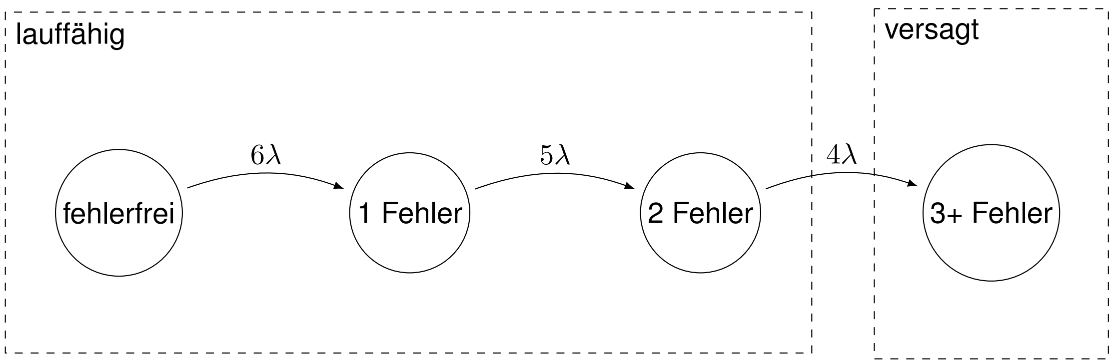
Diese Gleichung ist für alle $\mathbb{R} \setminus \{0,1\}$ wahr:



Demnach ist die Entscheidung, dass System A zuverlässiger ist als System B für alle Zuverlässigkeiten $R \in (0, 1)$ wahr.

3 AUFGABE 3

Q1: Markov Model ohne Reperatur:

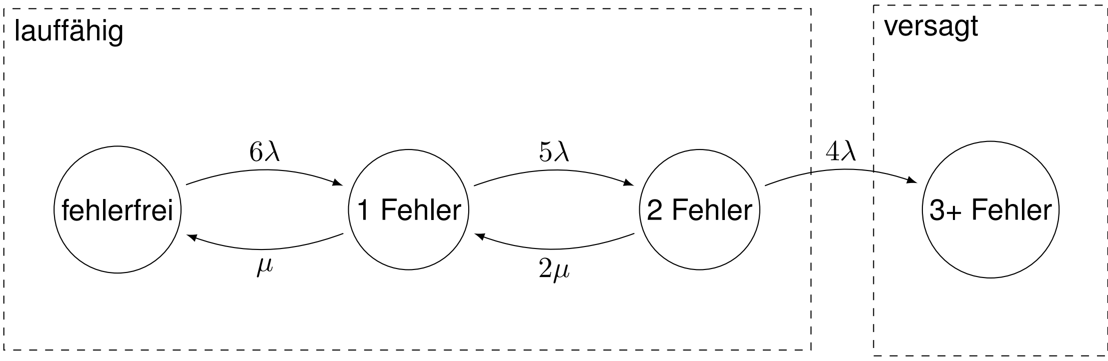


Bonus

- Werte für R und F:
  $F = 1,77467 \cdot 10^{-09}$
  $R = 1 - F \approx 0,9999999982$
- Model Datei:

```
[num states]
4
0 1 a failure
1 2 b failure
2 3 c failure
[assign]
a=6*lam
b=5*lam
c=4*lam
lam=0.000001
[END]
```

Q2: Markov Model mit Reperatur:



Bonus

- Werte für R und F:
  $F = 8,68786 \cdot 10^{-14}$
  $R = 1 - F \approx 1$
- Model Datei:

```
[num states]
4
0 1 a failure
1 0 d repair
1 2 b failure
2 1 e repair
2 3 c failure
[assign]
a=6*lam
b=5*lam
c=4*lam
d=mu
e=2*mu
lam=0.000001
mu=0.5
[END]
```